NORTH SYDNEY GIRLS HIGH SCHOOL

HSC Mathematics Extension 2

Assessment Task 1 Term 4 2012

Name:____**Mathematics Class:**_______

Student Number ____________________________

Time Allowed: **50 minutes + 2 minutes reading time**

Available Marks: **45**

Instructions:

Question 1 **(a) Multiple choice (10 marks)**

- Indicate your answer by colouring the appropriate circle

(b) Constrained answer (marks)

- Indicate your answer by entering it into the diagram on the answer sheet provided

Question 2 **Free response (10 marks)**

- Write your answers on the examination booklet provided
- Write on one side of the page only
- Do not work in columns
- Attempt all questions
- Show all necessary working
- Marks may be deducted for incomplete or poorly arranged work

Question	1 -10	11a	11 bc	12	Total	
E3	/10	/5			/15	
E2				/10	/10	
E9			/		/	
					/	%

Question 1

Multiple Choice (10 marks). Please answer on the answer sheet provided.

(1) Let $z = 5 - 2i$ and $w = 3 + 4i$. Then $z - \overline{w}$ is

(A) $2 - 6i$ (B) $2 + 2i$ (C) $8 - 6i$ (D) $2 - 2i$

(2) $\dfrac{1}{i^{101}} =$

(A) 1 (B) -1 (C) i (D) $-i$

(3) If $z = a + ib$, where $a \neq 0$ and $b \neq 0$, which of the following statements is false?

(A) $z - \overline{z} = 2bi$ (B) $|z| + |\overline{z}| = |z + \overline{z}|$

(C) $\arg(z) + \arg(\overline{z}) = 0$ (D) $|z|^2 = z\overline{z}$

(4) If $z = i - \sqrt{3}$ then $\arg(z^4)$ is

(A) $\dfrac{2\pi}{3}$ (B) $\dfrac{\pi}{3}$

(C) $-\dfrac{2\pi}{3}$ (D) $-\dfrac{\pi}{3}$

(5) The non-real roots of $z^3 + 8 = 0$ have

(A) modulus 8 and arguments $\pm\dfrac{2\pi}{3}$ (B) modulus 8 and arguments $\pm\dfrac{\pi}{3}$

(C) modulus 2 and arguments $\pm\dfrac{2\pi}{3}$ (D) modulus 2 and arguments $\pm\dfrac{\pi}{3}$

(6) If $z = 2i$ and $w = 2 - 2i$, then the modulus and argument of $\dfrac{z}{w}$ are respectively

(A) $\dfrac{1}{\sqrt{2}}, \dfrac{3\pi}{4}$ (B) $\dfrac{1}{\sqrt{2}}, -\dfrac{\pi}{4}$

(C) $\sqrt{2}, \dfrac{3\pi}{4}$ (D) $\sqrt{2}, \dfrac{\pi}{4}$

(7) Let $u = 4(\cos\frac{3\pi}{4} + \sin\frac{3\pi}{4})$ and $v = r(\cos\theta + i\sin\theta)$.

If $uv = 12(\cos\frac{\pi}{3} + i\sin\frac{\pi}{3})$, then

(A) $r = 3$, $\theta = \frac{5\pi}{12}$ (B) $r = 8$, $\theta = -\frac{5\pi}{12}$

(C) $r = 8$, $\theta = \frac{5\pi}{12}$ (D) $r = 3$, $\theta = -\frac{5\pi}{12}$

(8) The point R represents the complex number z on the Argand diagram. Which of the following describes the locus of R specified by $|z-6| = |z|$?

(A) Circle centre $(6,0)$, radius $|z|$

(B) Circle centre the origin, radius $|z|$

(C) Perpendicular bisector of $(0,0)$ and $(6,0)$

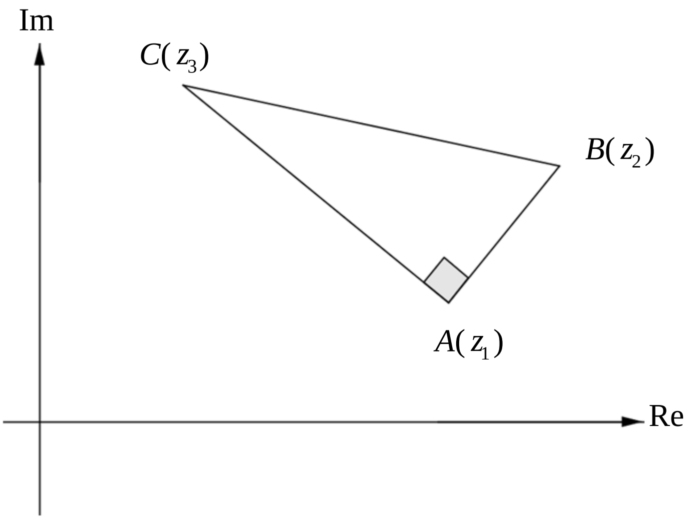
(D) Perpendicular bisector of $(0,0)$ and $(0,6)$

(9) The equation $x^2 - 2ix + 3 = 0$ has

(A) no roots (B) one real and one complex root

(C) two purely imaginary roots (D) two real roots

(10) The points *A*, *B* and *C* are three points on the Argand plane representing the complex numbers z_1, z_2 and z_3 respectively as shown on the diagram. *AB* is perpendicular to *AC*. Which statement is correct?



(A) $\arg\left(\frac{z_3 - z_2}{z_3 - z_1}\right) = \frac{\pi}{2}$

(B) $\arg\left(\frac{z_3 - z_1}{z_2 - z_1}\right) = \frac{\pi}{2}$

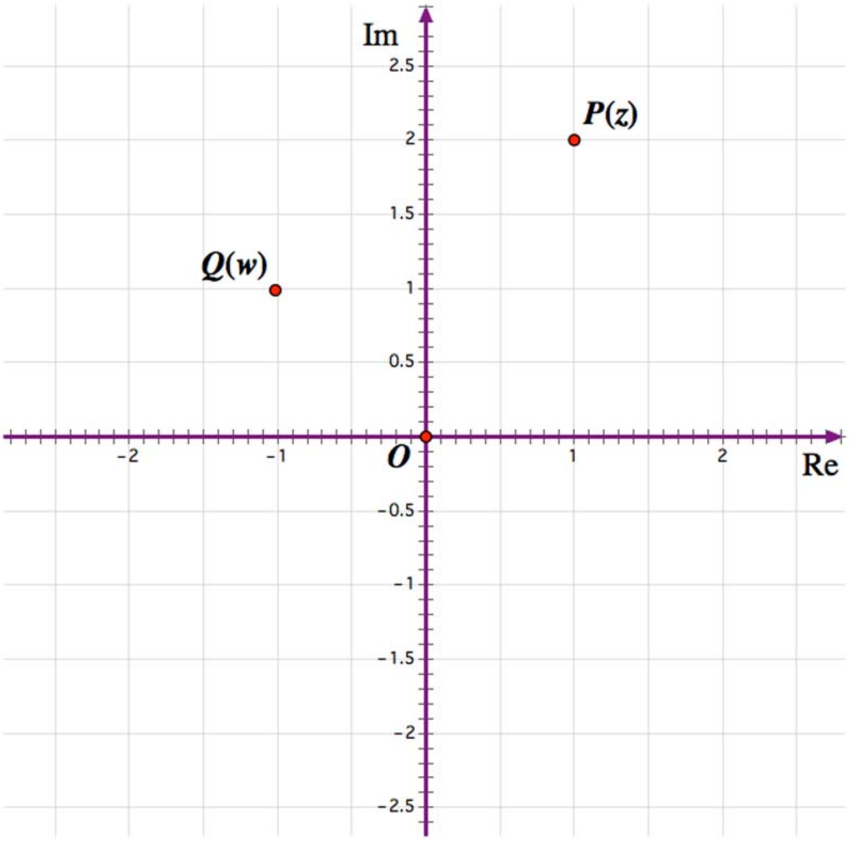
(C) $\arg[(z_3 - z_1)(z_2 - z_1)] = \frac{\pi}{2}$

(D) $\arg[(z_3 - z_2)(z_3 - z_1)] = \frac{\pi}{2}$

Question 11 continued: Constrained Answers (20 marks) .

Indicate your answer by entering it into the appropriate diagrams on the answer sheet provided.

(a) Point P represents the complex number z and Q represents the complex number ω.



Plot the points that represent **5**

(i) $A(\overline{z})$

(ii) $B(-\omega)$

(iii) $C\left(\frac{1}{\omega}\right)$

(iv) $D(iz)$

(v) $E(\omega - z)$

(b)

(i) Plot the roots of $z^8 - 1 = 0$ on the Argand Plane provided. **2**

(ii) Let the root with the smallest positive argument be α.

(iii) Write the complex number that is in the fourth quadrant in terms of α. **1**

(iv) Write $\overline{\alpha^{-10}}$ in modulus argument form. **1**

(c) Sketch the following loci and the answer sheet provided show all important features:

(i) $|z-3-2i|=|z+3|$ **2**

(ii) $-\frac{\pi}{4}<\arg(z)<\frac{\pi}{4}$ and $\text{Re}(z)\leq 2$ **2**

(iii) $0\leq\arg(z-5+2i)\leq\frac{\pi}{2}$ and $|z-5+2i|\leq 1$ **3**

(iv) $|z-3+4i|=2$ and determine the maximum and minimum values for $|z|$ **3**

Question 12: (10 marks) Free Response **Marks**

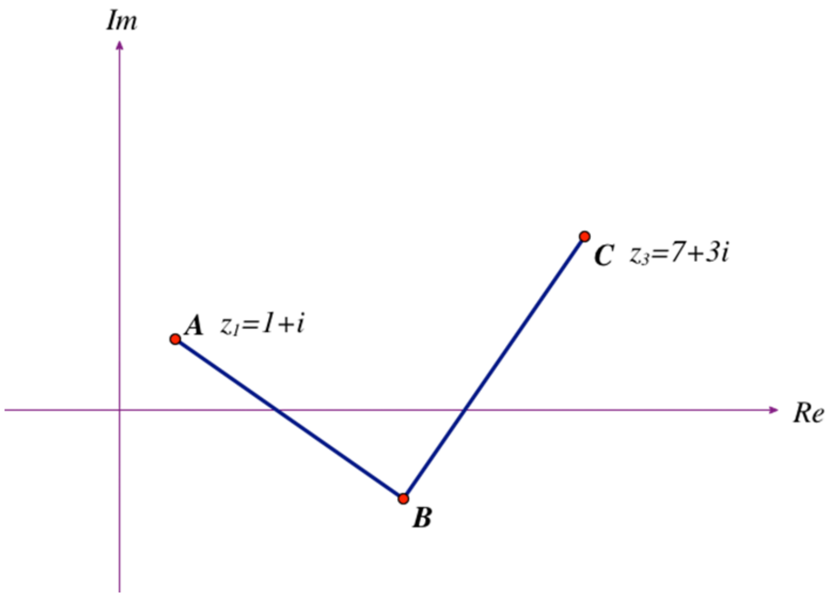
(a) Show that for any non zero complex number z **2**

$$\arg\left(\frac{z}{\bar{z}}\right)=2\arg(z)$$

(b) (i) Find the square roots of $5-12i$. **2**

(ii) Hence solve the equation $z^2-(1-4i)z-(5-i)=0$, expressing your answers in the form $a+ib$. **2**

(c)



The points A and C represent the complex numbers $z_1=1+i$ and $z_3=7+3i$ and B represents the complex number z_2. Find the complex number z_2 in the form $a+ib$ where a and b are real, such that ΔABC is isosceles and right-angled at B. **4**

END OF TEST

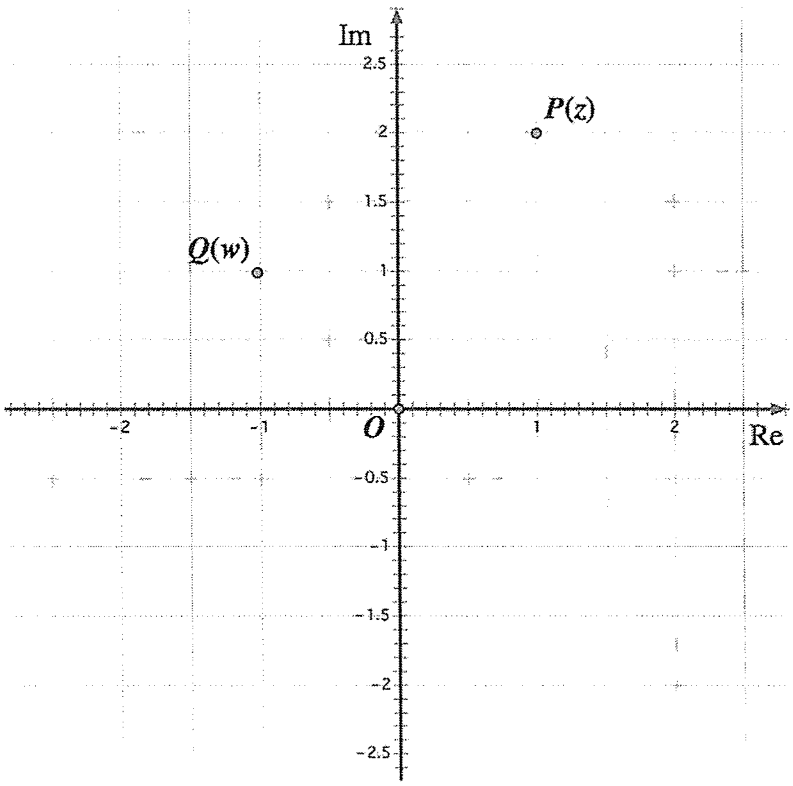
\Student Number:______________________ Mathematics Class: __________

Multiple choice answer sheet – Use pencil to completely colour the circle representing your answer.

1.	Ⓐ	Ⓑ	Ⓒ	Ⓓ	6.	Ⓐ	Ⓑ	Ⓒ	Ⓓ
2.	Ⓐ	Ⓑ	Ⓒ	Ⓓ	7.	Ⓐ	Ⓑ	Ⓒ	Ⓓ
3.	Ⓐ	Ⓑ	Ⓒ	Ⓓ	8.	Ⓐ	Ⓑ	Ⓒ	Ⓓ
4.	Ⓐ	Ⓑ	Ⓒ	Ⓓ	9.	Ⓐ	Ⓑ	Ⓒ	Ⓓ
5.	Ⓐ	Ⓑ	Ⓒ	Ⓓ	10.	Ⓐ	Ⓑ	Ⓒ	Ⓓ

Constrained Answer Sheet – Write your answer in the space provided.

11 (a)



Plot the points that represent

(i) $A(\overline{z})$

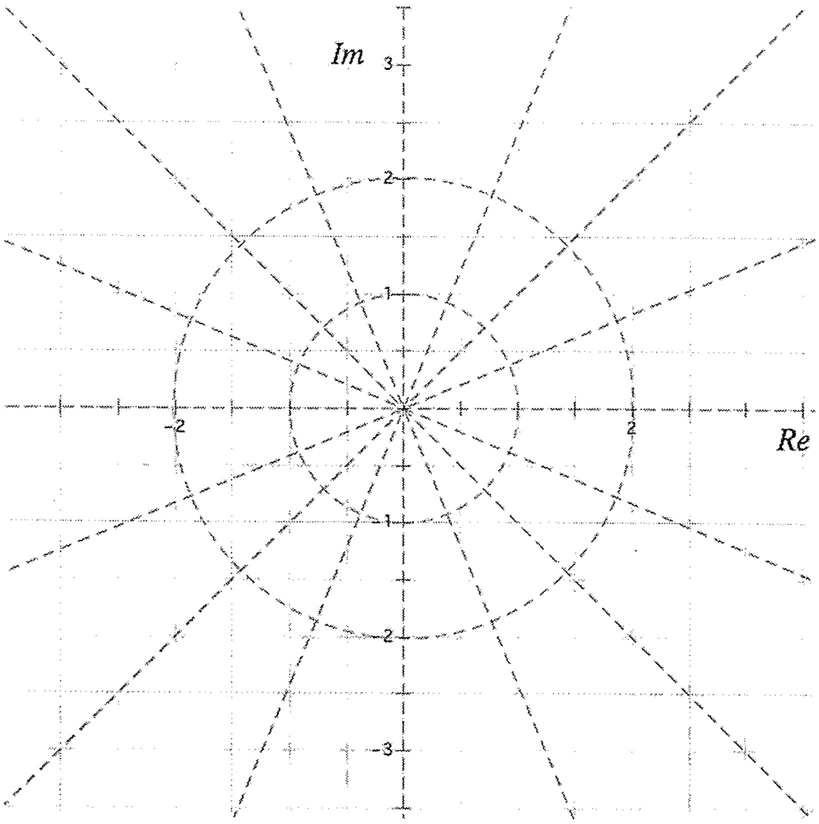
(ii) $B(-\omega)$

(iii) $C\left(\dfrac{1}{\omega}\right)$

(iv) $D(iz)$

(v) $E(\omega - z)$

(b) $z^8 - 1 = 0$



(iii) Write the complete number that is in the fourth quadrant in terms of α

(iv) Write $\overline{\alpha^{-10}}$ in modulus argument form.

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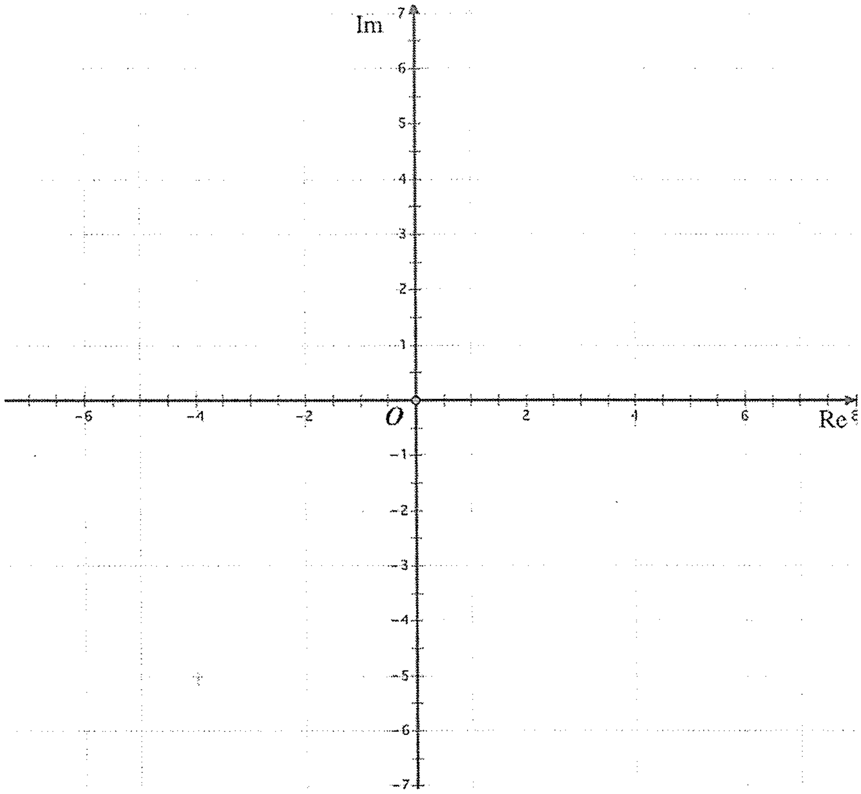
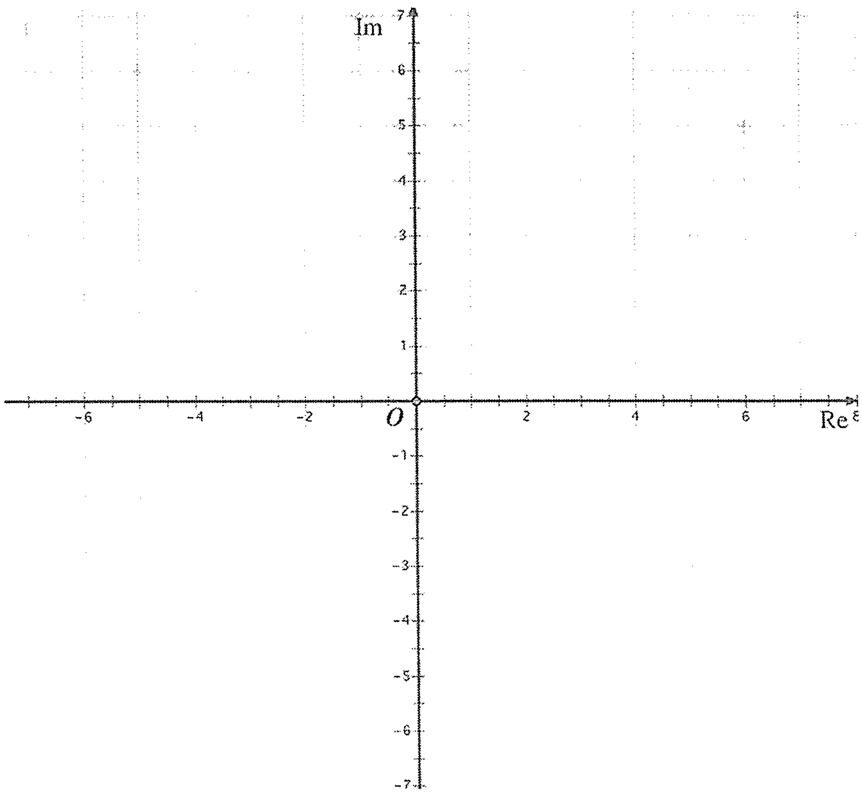
11.(c)

(i) $|z-3-2i|=|z+3+2i|$

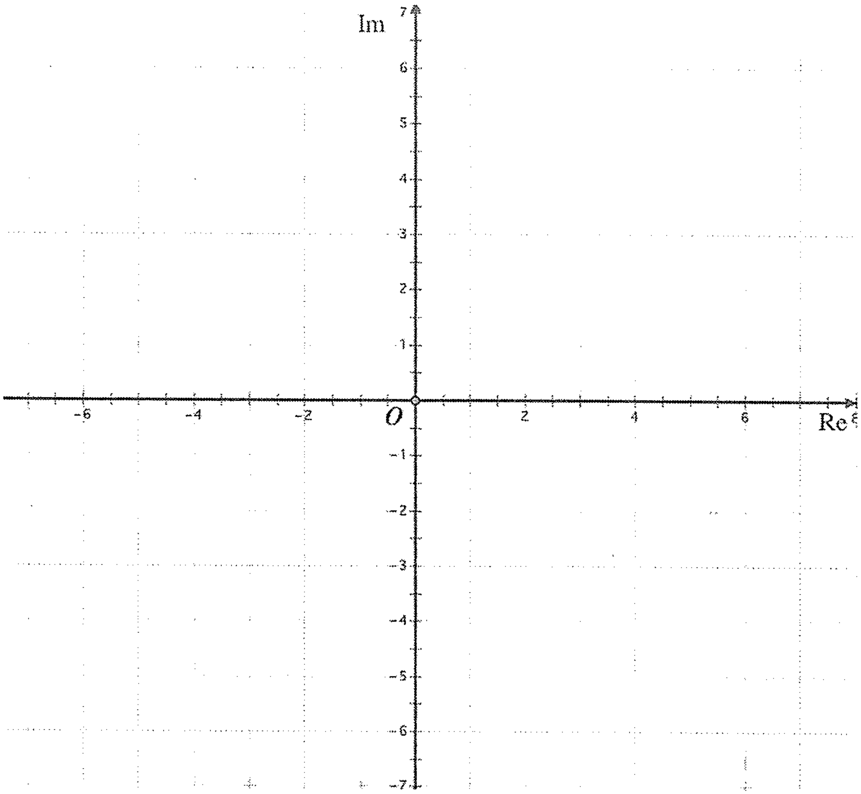


(ii) $-\frac{\pi}{4}<\arg(z)<\frac{\pi}{4}$ and $\mathrm{Re}(z)\leq 2$

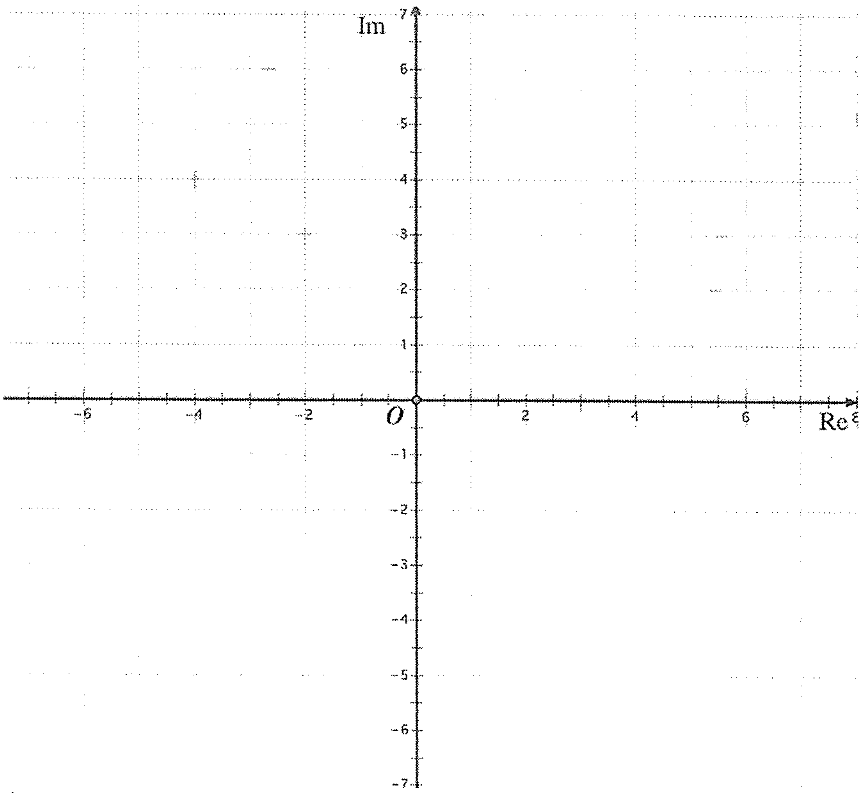


11.(c)

(iii) $0 \le \arg(z-2+2i) \le \frac{\pi}{2}$ and $|z-2+2i| \le 2$



iv) $|z-3+4i| = 2$



Maximum $|z|$ ____________

Minimum $|z|$ ____________

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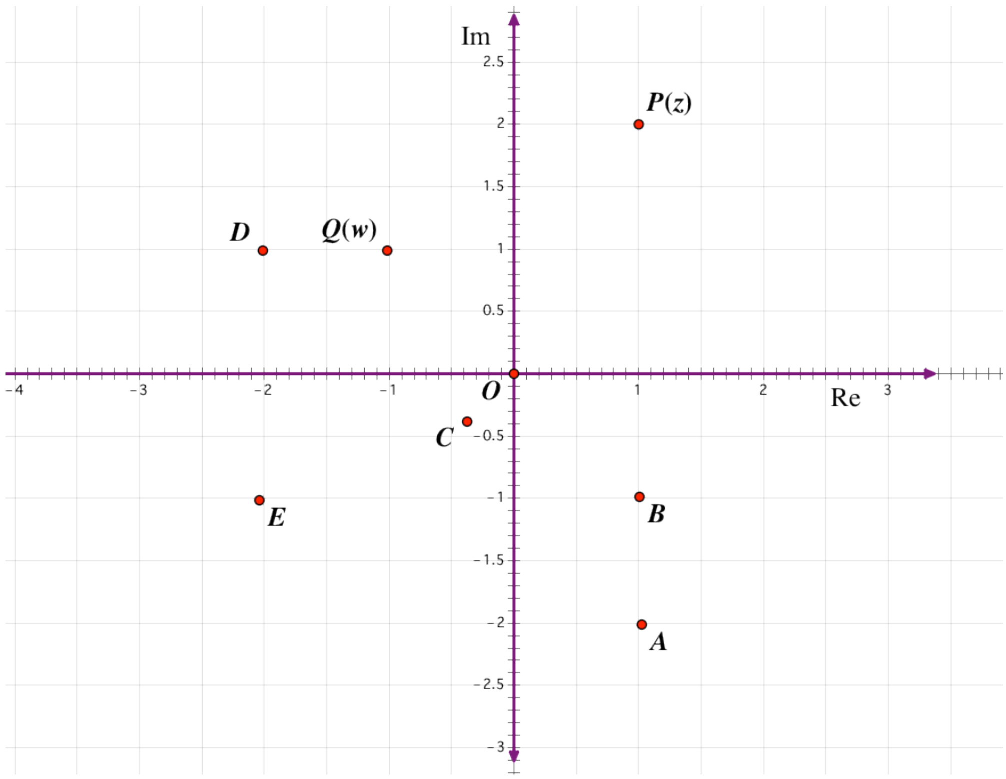
Multiple choice answer sheet – Use pencil to completely colour the circle representing your answer.

1.	Ⓐ	**B** (filled)	Ⓒ	Ⓓ	6.	**A** (filled)	Ⓑ	Ⓒ	Ⓓ
2.	Ⓐ	Ⓑ	Ⓒ	**D** (filled)	7.	Ⓐ	Ⓑ	Ⓒ	**D** (filled)
3.	Ⓐ	**B** (filled)	Ⓒ	Ⓓ	8.	Ⓐ	Ⓑ	**C** (filled)	Ⓓ
4.	Ⓐ	Ⓑ	**C** (filled)	Ⓓ	9.	Ⓐ	Ⓑ	**C** (filled)	Ⓓ
5.	Ⓐ	Ⓑ	Ⓒ	**D** (filled)	10.	Ⓐ	**B** (filled)	Ⓒ	Ⓓ

Constrained Answer Sheet – Write your answer in the space provided.

11 (a)



Plot the points that represent

(i) $A(\bar{z})$

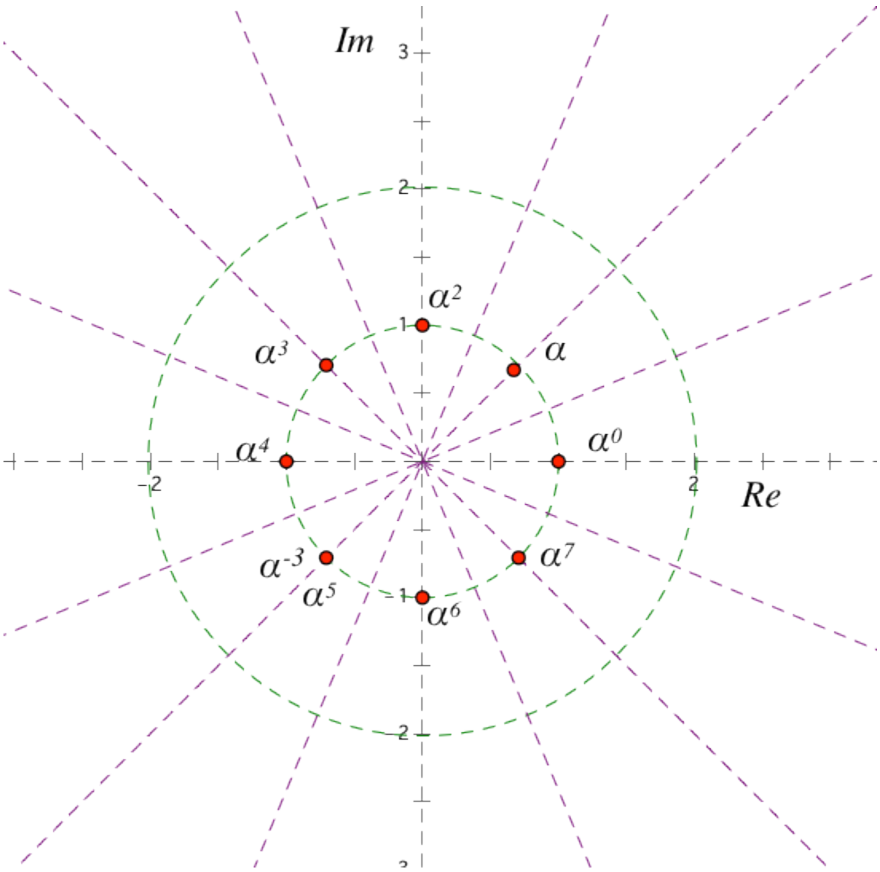
(ii) $B(-\omega)$

(iii) $C\left(\dfrac{1}{\omega}\right)$

(iv) $D(iz)$

(v) $E(\omega - z)$

(b) $z^8 - 1 = 0$



(iii) If α is the complex root in quadrant 1, write the complex root in quadrant 4 as a power of α.

α^7 or α^{-1}

(iv) Write $\overline{\alpha^{-10}}$ in modulus argument form.

$\overline{\alpha^{-10}} = \overline{\alpha^6} = a^2 = \cos\left(\frac{\pi}{2}\right) + i\sin\left(\frac{\pi}{2}\right)$

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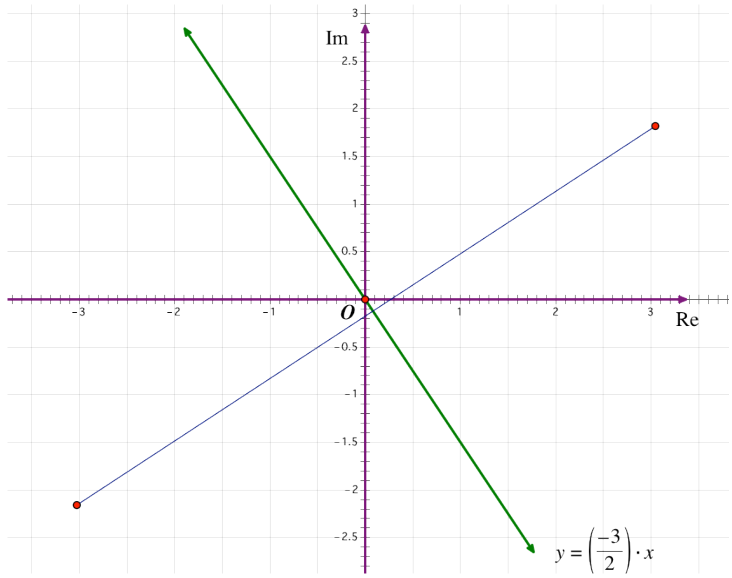
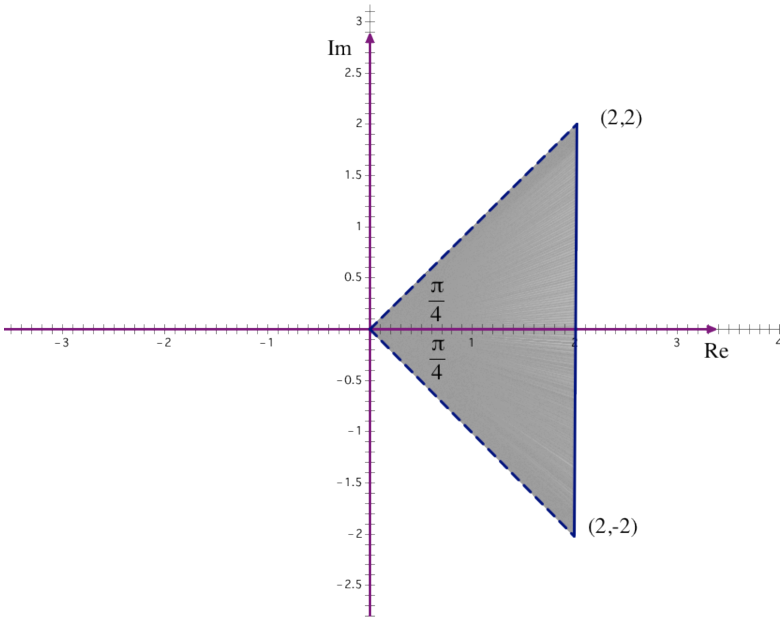
11.(c)

(i) $|z-3-2i|=|z+3+2i|$

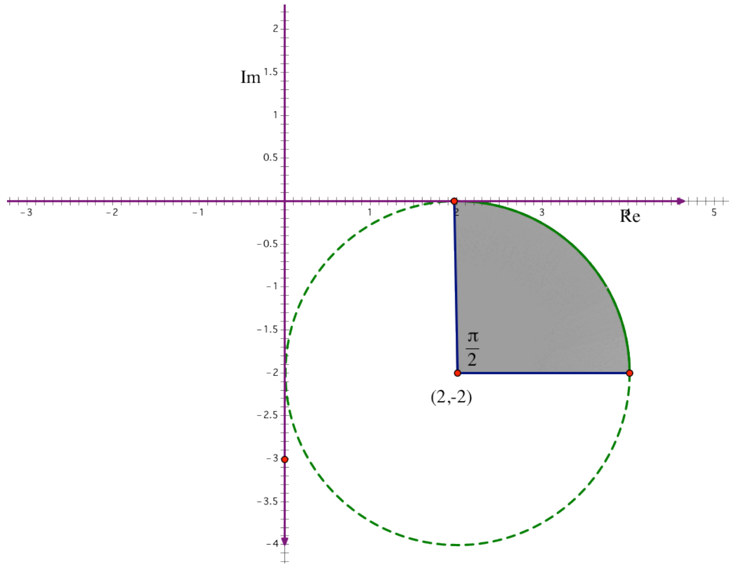


(ii) $-\frac{\pi}{4}<\arg(z)<\frac{\pi}{4}$ and $\mathrm{Re}(z)\le 2$

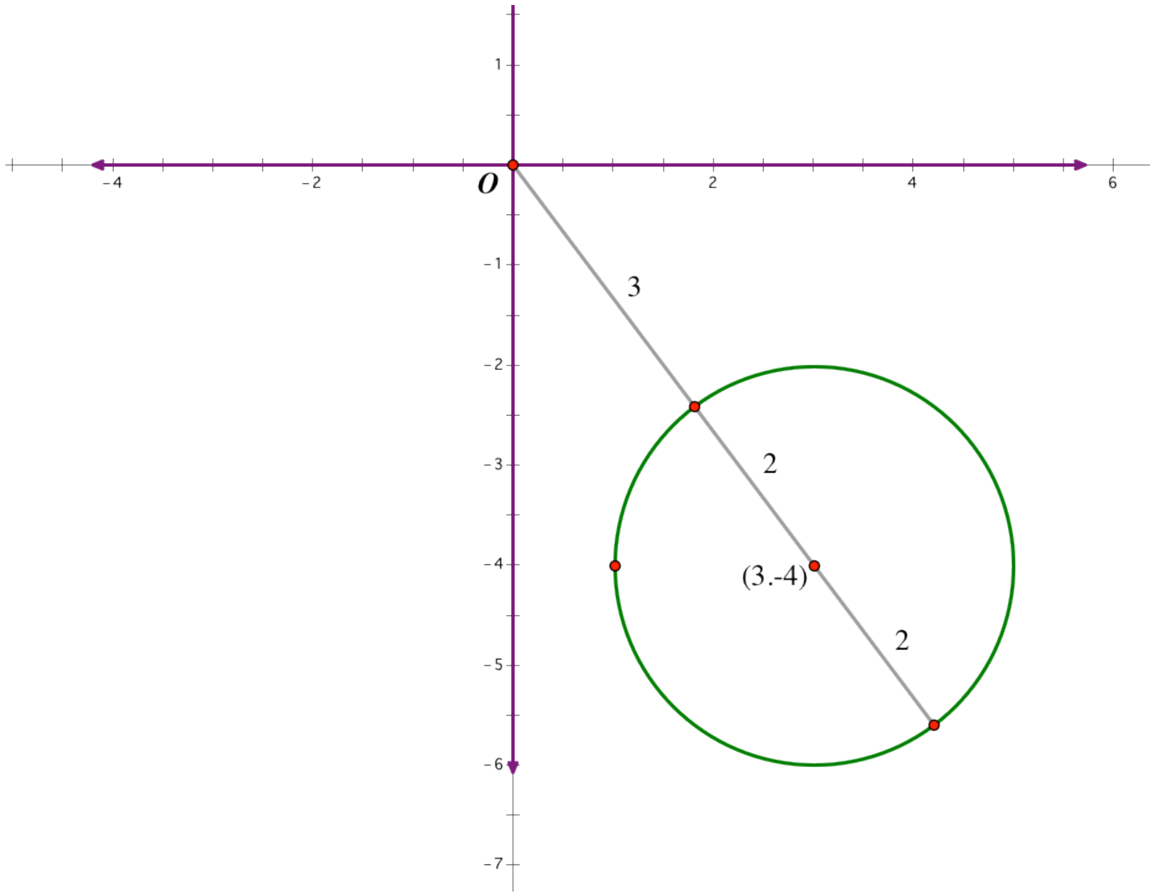


11.(c)

(iii) $0 \le \arg(z-2+2i) \le \frac{\pi}{2}$ and $|z-2+2i| \le 2$



iv) $|z-3+4i| = 2$



Maximum $|z|$ ______=7

Minimum $|z|$ ______=3

Question 12: (10 marks) Solutions Free Response Marks

(a) Show that for any non zero complex number $\arg\left(\frac{z}{\overline{z}}\right) = 2\arg(z)$ **2**

Let $z = r\text{cis}\theta$ and so $\overline{z} = r\text{cis}(-\theta)$

$$\frac{z}{\overline{z}} = \frac{r\text{cis}\theta}{r\text{cis}(-\theta)} = \text{cis}(2\theta)$$

$$\therefore \arg\left(\frac{z}{\overline{z}}\right) = 2\theta = 2\arg z$$

(b)

(i) Find the square roots of $5-12i$.

$$5-12i = (x+yi)^2$$
$$= x^2 - y^2 + 2xyi$$

$$5 = x^2 - y^2$$
$$-6 = xy$$

$$\therefore x = -3 \text{ and } y = 2$$
$$\therefore x = 3 \text{ and } y = -2$$

Square roots of $5-12i$ are $-3+2i$ and $3-2i$

(ii) Hence solve the equation $z^2-(1-4i)z-(5-i)=0$, expressing your answers in the form $a+ib$. **2**

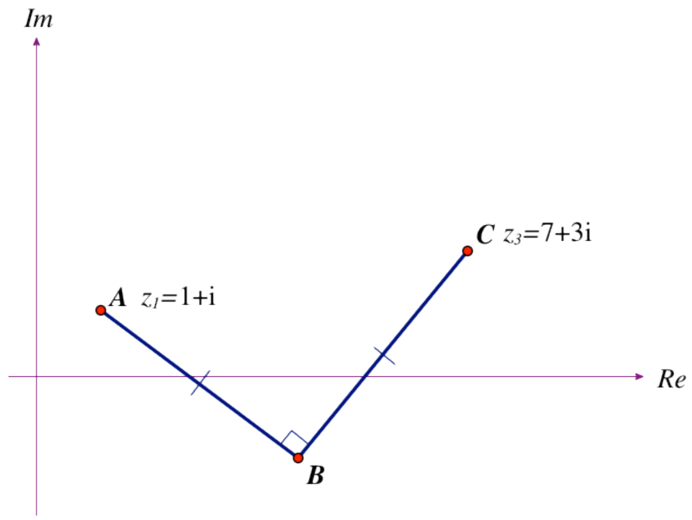
$$z = \frac{1-4i \pm \sqrt{\left(-(1-4i)\right)^2 - 4\times 1\times -(5-i)}}{2}$$

$$z = \frac{1-4i \pm \sqrt{5-12i}}{2}$$

$$z = \frac{1-4i \pm (3-2i)}{2}$$

$$z = \frac{4-6i}{2} = 2-3i \quad \text{and} \quad z = \frac{-2-2i}{2} = -1-i$$

(c)



The points *A* and *C* represent the complex numbers $z_1 = 1 + i$ and $z_3 = 7 + 3i$ and B represents the complex number z_2. Find the complex number z_2 in the form $a + ib$ where a and b are real, such that ΔABC is isosceles and right-angled at *B*. **4**

$$\vec{BA} = z_1 - z_2 = 1 + i - a - bi = 1 - a + i(1-b)$$

$$\vec{BC} = z_3 - z_2 = 7 + 3i - a - bi = 7 - a + i(3-b)$$

$$i \times \vec{BC} = \vec{BA}$$

$$i \times (7 - a + i(3-b)) = 1 - a + i(1-b)$$

$$i(7-a) - (3-b) = 1 - a + i(1-b)$$

By equating real and imaginary parts

$$-(3-b) = 1 - a$$

$$b = 4 - a$$

and

$$(7-a) = (1-b)$$

$$7 - a = 1 - (4-a)$$

$$7 - a = -3 + a$$

$$10 = 2a$$

$$a = 5$$

$$b = 4 - a$$

$$b = 4 - 5$$

$$b = -1$$

$$\therefore z_2 = 5 - i$$